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YET ANOTHER QUANTUM MINDPUZZLE: TELEPORTATION OF A QUANTUM STATE

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Quantum mechanics (QM) is famous for its mindpuzzling problems which were mindpuzzling even to its founders [1,2], not to speak of thousands of students faced with a certain weirdness of QM. Tunneling effect, Feynman path integrals interpretation of QM, two slit experiment, Bohm-Aharonov effect, Einstein-Podolsky-Rosen (EPR) paradox [1] are only some of the weird stuff QM is full of. Where does this weirdness come from? Well, almost all paradoxes (it of course depends on what you call a paradox...) of QM which stood the test of time, come from the non-locality of a wave function which contains all of the physics of a certain problem. This effectively means that if you mess with the wave function in a particular range of space variables, you will simultaneously affect the wave function in all the rest of the space. Teleportation of a quantum state is also based on this "mechanism" of QM and is, to a certain extent, also paradoxical. Niels Bohr once said: *"Anyone who is not shocked by quantum theory has not understood it."* Following this line of reasoning, we will try to shock you once again, hoping to increase your understanding of a weird (sometimes called spooky) nature of QM.

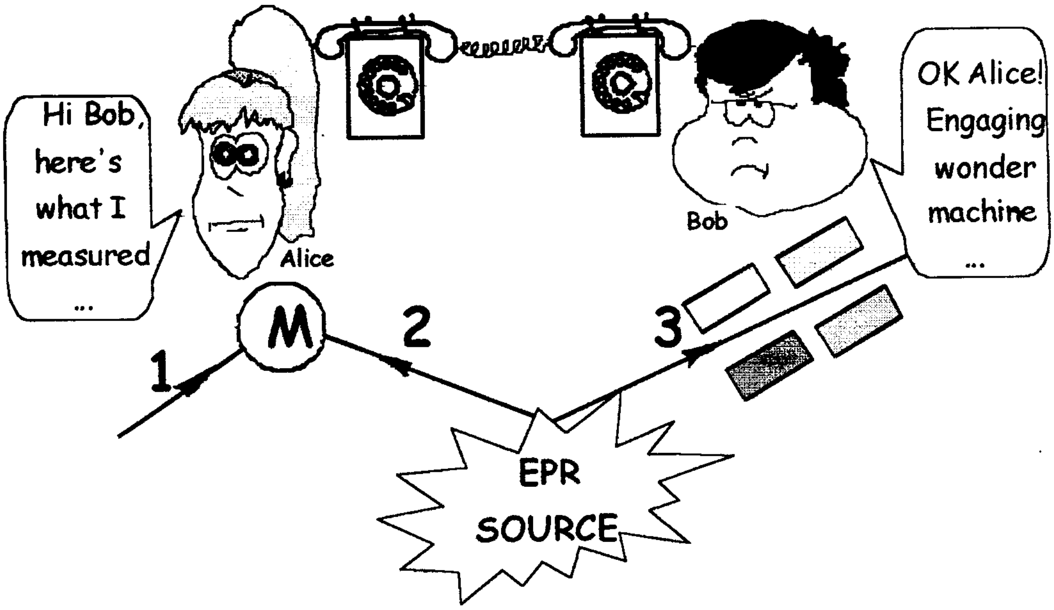

Figure 1: Alice and Bob trying to teleport the state of particle 1 to particle 3

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The "gedanken" (and not only gedanken. see [4]) experimental setup drawn in Fig. 1. Alice and Bob (the names come from A and B - the last experimentalist would be called e.g. Zane) are two keen experimentalists trying to teleport a state of a particle 1 on distance. i.e. Bob would like to produce particle 3 in the same quantum state as that of the particle 1. Let us confine ourselves to the fermion particles. The states we will be interested in are going to be their spin states. Particle 1 has some *unknown* spin state. Written in the basis, which makes the z-component of a spin operator (S_z) diagonal, it would look like (quite general)

$$|1\rangle = a|\uparrow_1\rangle + b|\downarrow_1\rangle \; , \tag{1}$$

where $|a|^2 + |b|^2 = 1$. Alice and Bob are connected through the *EPR* source, which produces two particles, 2 and 3 in a singlet-spin state, which can be written in the same basis as:

$$|\Psi_{23}{}^{s}\rangle = \frac{1}{\sqrt{2}}(|\uparrow_2\rangle|\downarrow_3\rangle - |\downarrow_2\rangle|\uparrow_3\rangle) . \tag{2}$$

Alice gets particle 2 and Bob gets particle 3. Alice then performs the measurement on both particles she has, namely 1 and 2 (point denoted by M in Fig. 1). She has a very specific apparatus which can read one of the four possible combinations of 1 and 2 spin states, namely:

$$\begin{aligned} |\Psi_{12}{}^{\pm}\rangle &= \frac{1}{\sqrt{2}}(|\uparrow_1\rangle|\downarrow_2\rangle \pm |\downarrow_1\rangle|\uparrow_2\rangle) \\ |\Phi_{12}{}^{\pm}\rangle &= \frac{1}{\sqrt{2}}(|\uparrow_1\rangle|\uparrow_2\rangle \pm |\downarrow_1\rangle|\downarrow_2\rangle) \end{aligned} \tag{3}$$

This means that Alice projects the combined state of 1 and 2 particles onto a basis of states (3), specific of her measurement apparatus. This basis of states is known as Bell's basis (you can probably smell it by now - things are going to get weird). Then she phones Bob and tells him which combination she measured. Bob engages one of his four wonder machines (denoted by differently colored rectangles in Fig. 1). This machine acts on particle 3 in such a way that it changes its state to the state of particle 1 had before Alice measured it (1). Wonderful - or full of wonder you might say! How can it be? Note that Alice does not know the spin state of particle 1, and neither does Bob. The key to this apparent paradox (or true paradox if you wish) lies in the fact that we always have to concentrate on a wave function of the *whole* system. Knowing wave function of the spin-singlet state of particles 2 and 3 and the wave function of particle 1 (1) (which has the unknown coefficients *a* and *b*), we can write the complete wave function (which includes all of the particles in the system) as:

$$\begin{aligned} |\Psi\rangle_{123} &= \frac{1}{\sqrt{2}}(a|\uparrow_1\rangle + b|\downarrow_1\rangle) \otimes (|\uparrow_2\rangle|\downarrow_3\rangle - |\downarrow_2\rangle|\uparrow_3\rangle) \\ &= \frac{1}{2}\{|\Psi^{+}{}_{12}\rangle \otimes (-a|\uparrow_3\rangle + b|\downarrow_3\rangle) + |\Psi^{-}{}_{12}\rangle \otimes (-a|\uparrow_3\rangle - b|\downarrow_3\rangle) + \\ &\quad |\Phi^{+}{}_{12}\rangle \otimes (-b|\uparrow_3\rangle + a|\downarrow_3\rangle) + |\Phi^{+}{}_{12}\rangle \otimes (b|\uparrow_3\rangle + a|\downarrow_3\rangle)\}, \end{aligned} \tag{4}$$

where we have written it in this special way which corresponds to what Alice can measure for particles and 2. Now, when Alice performs the measurement, she projects the state of 1 and 2 particles into one four possible combinations (3). But, this also means that she projects the state of particle 3 in one of possible ways, namely

$$\begin{aligned}&-a|\uparrow_3\rangle+b|\downarrow_3\rangle,\\&-a|\uparrow_3\rangle-b|\downarrow_3\rangle,\\&-b|\uparrow_3\rangle+a|\downarrow_3\rangle,\\&+b|\uparrow_3\rangle+a|\downarrow_3\rangle.\end{aligned} \tag{5}$$

Alice knows what she has done, and she tells it to Bob. All he has to do is to unitary transform the state of particle 3 (5) in order to get it in the state written in equation (1), since the state of a particle 3 now becomes "*a*" and “*b*”-marked! The following matrices represent the actions of his four wonder machine (again in the S_z-diagonal basis):

$$\begin{bmatrix}-1 & 0\\0 & 1\end{bmatrix},\begin{bmatrix}-1 & 0\\0 & -1\end{bmatrix},\begin{bmatrix}0 & 1\\-1 & 0\end{bmatrix},\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}. \tag{6}$$

And that's how quantum teleportation works.

The paradox of quantum teleportation lies in the fact that Alice's measurement acting only on particles 1 and 2, influences the state of particle 3) which can be miles away [3]. This is also the basis on which the famous EPR paradox works - non-locality of the wave function. What is even stranger is that particles 1 and 3 never interacted! The possibility of teleportation also crucially depends on how Alice performs her measurement. The choice of the Bell's basis for Alice's measurement was essential for a successful teleportation. This extremely interesting physics is currently getting its firm experimental confirmation [3,4] and much of theoretical work is ongoing, promising sometimes "Star Trek"-like future, especially when it concerns teleportation of continuous quantum variables [5,6]. The story we presented opens a waste area for philosophical discussions, but we prefer to leave it for coffee or beer small talks.

The authors sincerely hope they have managed to shock you successfully with the nature of QM once more. You know what Mr. Bohr said...

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